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Are We Approaching an Economic Singularity? Information Technology and the Future of Economic Growth†

By William D. Nordhaus*

What are the prospects for long-run economic growth? One prominent line of economic thinking is the trend toward stagnation. Stagnationism has a long history in economics, beginning prominently with Malthus and occasionally surfacing in different guises. Prominent themes here are the following: Will economic growth slow and perhaps even reverse under the weight of resource depletion? Will overpopulation and diminishing returns lower living standards? Will unchecked CO_2 emissions lead to catastrophic changes in climate and human systems? Have we depleted the store of potential great inventions? Will the aging society lead to diminished innovativeness? (*JEL* D83, E25, O31, O32, O41, O47)

There is a vast literature on the potential sources of stagnation. In the modern era, the "Limits to Growth" school was an early computerized modeling effort that produced scenarios for overshoot and decline in living standards (see Meadows et al. 1972; Meadows, Meadows, and Randers 1992). In his economic history of the United States, Gordon (2016) argued that a decline in fundamental inventions might slow growth. Some foresee a long period of demand-side stagnation in the wake of the long recession that began in 2008 (see Summers 2014), although this looks less plausible for the United States in 2019 given the strong economic expansion.

However, the present study looks at the opposite idea, a recently launched hypothesis that I label the *Singularity*. The idea here is that rapid growth in information technology and artificial intelligence will cross some boundary, after which economic growth will rise rapidly as an ever-increasing pace of improvements cascade through the economy. The most prominent exponents are computer scientists (see the next section for a discussion and references), but a soft version of this theory has recently been advanced by some economists as well (Brynjolfsson and McAfee 2014, Varian 2016). Even some business research firms like Accenture have jumped on the bandwagon, predicting doubling of growth over the next two decades from artificial intelligence.

*Cowles Foundation, Yale University, PO Box 208268, Yale Station, New Haven, CT 06511 (email: william.nordhaus@yale.edu). Richard Rogerson was coeditor for this article. The author is grateful for comments from Ben Jones, Hal Varian, and anonymous referees. The author declares no financial conflict.

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The purpose of this study is twofold. First, I lay out some of the history, current views, and analytical basis for rapidly rising economic growth. Next, I propose several diagnostic tests that might determine whether Singularity is occurring and apply these tests to recent economic behavior in the United States. The tentative conclusion is that the Singularity is not near, but we have developed tests that can give early warning signs of its occurrence.

I. Artificial Intelligence and the Singularity

For those with a background primarily in economics, the present section is likely to resemble economic science fiction. It will explain the history and a modern view about how the rapid improvements in computation and artificial intelligence (AI) have the potential to increase their productivity and breadth to the extent that human labor and intelligence will become superfluous. The standard discussion in computer science has no explicit economic analysis and leaves open important economic issues that will be addressed in later sections.

It will be useful to summarize the argument before giving further background. The productivity of computers and software has grown at phenomenal rates for more than a half-century, and rapid growth has continued up to the present. Developments in machine learning and artificial intelligence are taking on an increasing number of human tasks, moving from calculations to search to speech recognition, psychotherapy, and autonomous activities on the road and battlefield. At the present growth of computational capabilities, some have argued, information technologies will have the skills and intelligence of the human brain itself. For discussions of the background and trends, see Moravec (1988), Kurzweil (2000, 2005), and Schmidt and Cohen (2013).

A. *The Progress of Computing*

The foundation of the accelerationist view is the continuing rapid growth in the productivity of computing. One measure of productivity is the cost of computing, shown in Figure 1. The constant-dollar costs of a standard computation have declined at an average annual rate of 53 person per year over the period 1940–2014. There may have been a slowing in the speed of chip computations over the last decade, but the growth in parallel, cloud, and high-performance clusters, as well as improvements in software, appears to have offset the slowing of hardware speed for many applications.

Computer scientists project the trend shown in Figure 1 into the indefinite future. At some point, these projections move from computer science to computer science fiction. They involve improved conventional devices and eventually, quantum computing. If high-qubit quantum computing becomes feasible, this is likely to increase computation speeds by a factor of 100 million or more according to Google. This is about four decades of advances at the rates of recent years. If large-scale quantum computing is available, then the constraints on artificial intelligence will largely be ones of software and engineering (see Moravec 1988, Kurzweil 2005).

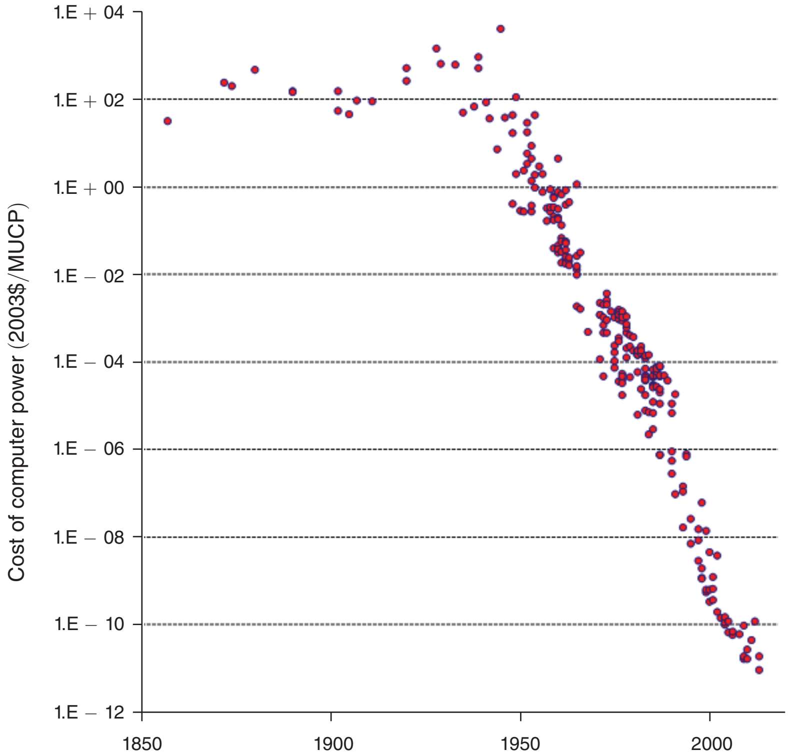

Figure 1. The Progress of Computing Measured in Cost per Computation per Second Deflated by the Price Index for GDP in 2006 Prices

Source: Nordhaus (2008)

One important milestone will come when computers attain the capacity of the human brain. Current estimates are that the computational capacity of the human brain is in the range of 10^{18} computations per second or petaflops ("flops" are floating point operations per second). This benchmark was reached in 2018 by the Oak Ridge Summit supercomputer. The speed of supercomputers has been growing at a rate of 63 percent per year over the 2009–2019 period (Top500, 2019). Computational speed does not easily translate into human intelligence, but it would provide the raw material for scientists to work with.

B. *From Computing to Singularity*

As computer scientists look further into their crystal ball, they foresee artificial intelligence moving toward superintelligence, which denotes "intellect that is much smarter than the best human brains in practically every field, including scientific creativity, general wisdom and social skills." (Bostrum 2006, 11).

At the point where computers have achieved superintelligence in all human activities, we have reached the "Singularity" where humans become economically superfluous in the sense that their relative performance is negligible. Humans would be the equivalent of oxen as compared to supertankers. Superintelligent computers are the last invention humans would make, as described by the mathematician Irving Good as follows:

> Let an ultraintelligent machine be defined as a machine that can far surpass all the intellectual activities of any man however clever. Since the design of machines is one of these intellectual activities, an ultraintelligent machine could design even better machines; there would then unquestionably be an "intelligence explosion," and the intelligence of man would be left far behind. Thus the first ultraintelligent machine is the last invention that man need ever make.
>
> —Good (1965, 33)

This point at which the rate and breadth of technological change will be so great is sometimes call the "Singularity" in a sense analogous to passing over the event horizon into a black hole—here the event horizon is where the forces of computer intelligence leave no room for human labor.

C. *Three Tasks of Intelligent Machines*

In considering the role of machine intelligence in the economy, it will be useful to distinguish three stages. These are calculations, control, and innovation.

Stage One: Calculation.—The first stage of machine intelligence is simply to calculate. We can think of this conceptually as reading some data as inputs and producing processed data as outputs. More generally, it also involves increasingly rapid and deep storage and communication as well. We take this stage largely for granted today because such a range and depth of calculations are occurring around us daily, but this stage was revolutionary in living memory. The progress of computing in Figure 1 shows how much progress has been made in simply transforming 0s and 1s. We should not overlook its importance, however. Complicated and sophisticated calculations are behind weather forecasts, medical scans, advanced econometrics, and much more.

Stage Two: Computerized Control and Production.—The first stage blends into the second stage when computers begin to control other machines and engage in production. The most dramatic examples are programmable robots, for example, ones that build cars or perform surgery. Less dramatic but probably more revolutionary are systems such as computer-assisted design, which harnesses computer algorithms to build and test visual models in areas from the design of ships to perfume bottles. Computer-assisted manufacturing is ubiquitous in areas as diverse as oil refining, electricity generation, airline reservation systems, and matching of medical residents. At the extreme of computerized control are autonomous systems such as cars that drive themselves or weapons that operate alone on the battlefield (discussed in the final section). Stage two is like stage one in that machines take orders, or in the language of economics follow the instructions in a production function. A key issue discussed below is whether computers can produce everything, and whether they can produce everything with vanishingly small amounts of human labor.

Stage Three: Computerized Innovation.—The final stage in computerized control involves computers innovating or designing new and improved production processes. In the language of economics, this involves designing new production functions. Innovation is the last frontier of intelligent or superintelligent machines in that they can not only produce everything that humans need (as in stage two) but can also design new and improved processes that lower production costs.

Computers are beginning to make inroads in stage three. An interesting example is automated theorem proving, which is proving theorems by computer programs. Computers are beginning to write poems and newspaper stories and compose songs. However, at present no examples come to mind where computers have, say, invented a new pharmaceutical, designed a new product, or found a new fundamental particle, but there seems no reason to doubt that these are possible.

It is clear that computer intelligence dominates task one of calculation, has begun to invade stage two of production and control, and has made very little progress on stage three's innovation. But all this is as of 2019. We must recall the history of predictions about computers. The best minds have continuously attempted to predict what computers cannot do, but it was only a matter of time before computers did it.

II. Historical Perspectives on Singularity in Economics

Before we find ourselves falling into the event horizon of accepting the Singularity hypothesis, we need to clarify some of the implicit economic assumptions that lie behind it. While the progress of computers shown in Figure 1 is astonishing, it does not address the central question of the impact of IT on the economy as a whole. The continued rapid growth of information technology has no necessary implication for aggregate economic growth. The reason is that the economy does not run on bits alone, either on the demand side or the supply side. Consumers may love their iPhones, but they cannot eat the electronic output. Production requires scarce material inputs ("stuff") in the form of labor, energy, and natural resources as well as information for all goods and services.

Singularity is a recent theory, but concerns about the displacement of humans by machines have been persistent for more than two centuries. The concerns tended to focus on the disappearance of particular jobs or occupational categories. With the rise of computers, the major concern has been the replacement of unskilled labor by computers and robots.

Macroeconomic concerns about rapid productivity growth and "automation," as it was called in the early days, focused first on the potential for the satiation of human wants and a crisis either of unemployment or superabundant leisure. This was the theme of J. M. Keynes's essay, "The Economic Prospects for Our Grandchildren" (1930, 358). He thought that with rapid technological growth, the problem of humanity would be "how to use his freedom from pressing economic cares, how to occupy the leisure, which science and compound interest will have won for him, to live wisely and agreeably and well." Although we are close to the hundred-year mark, there is no sign that humans have found themselves satiated with goods or leisure.

One of the earliest attempts to deal with the macroeconomic implications of computerization was Herbert Simon in "The Shape of Automation" (1965). Simon was unique in the intellectual history of the accelerationist debate in being a pioneering computer scientist as well as a leading economist. Writing a half-century ago, he was a self-described "technological radical." He wrote, "I believe that, in our time, computers will be able to do anything a man can do" (pp. xii–xiii). At the same time, he was not what I will call an accelerationist, holding that "computers and automation will contribute to a continuing, but not greatly accelerated, rise in productivity" (p. xiii). As we show below, it seems likely that if, as Simon believed, computers can do anything humans can do, then productivity would greatly accelerate.

Simon's pioneering analysis was simple, relying on what is known as the "factor price frontier." This is the concept that, under highly stylized conditions, factor rewards can be summarized by the equation

$$wa_L + (1 + r)a_K = 1.$$

In Simon's analysis, output is produced by labor and capital, there are constant returns to scale, and output is a homogenous product that can be used for either consumption or new capital. In this equation, $w =$ wage rate, $a_L =$ labor input coefficient, $a_K =$ capital input coefficient, and $r =$ real interest rate. The price of goods is normalized to one.

Simon correctly argues that technological change affects unit inputs by lowering the labor and/or capital input coefficients so that at existing factor prices, the cost of production with the new technology is less than 1. Using the notation of the factor-price equation (where subscripts 0 are original factor prices, and asterisks denote the new technology), with an innovation, $w_0 a_L^* + (1 + r_0)a_K^* < 1$. Under competition, factor prices will rise until the cost is equal to the price at 1, so in equilibrium, an innovative technology will raise either wage rates or interest rates or both.

Simon does not deploy a formal model for his critical next step. He argues that labor is inelastically supplied while capital is elastically supplied (so r is close to constant). This leads him to conclude that future changes in technology from automation will lead to nearly constant interest rates. He further argues for a near-constant share of capital in national income, which then implies that "almost all the increased productivity will go to labor" (p. 15).

Simon's pathbreaking analysis pointed to an important result about factor prices—that it is impossible in the neoclassical framework to have both a falling rate of profit and immiseration of the working classes (a formal analysis is in Samuelson 1957). However, his analysis was unable to deal with the potential of rapidly growing capital productivity in the case where the *share* of capital in national output is rising rather than stable.

There is much about robots but remarkably little writing on Singularity in the modern macroeconomic literature. While trend productivity growth has clearly risen from the period before the Industrial Revolution, the workhorse models today assume steady productivity and real income growth into the future. Zeira (1998) examined the implications of biased technological change with bounded growth. Olsen and Hémous (2014) examined a model of endogenous growth with automation. They showed an interesting pattern of wage growth for low-skilled workers. Acemoglu and Restrepo (2018) examined a growth model with automation that has a balanced growth path (more on this below). Similarly, Sachs, Benzell, and LaGarda (2015) analyzed a robotic economy with immiseration. They found that the rise of robots is more likely to lower the welfare of young workers and future generations when the saving rate is low, high- and low-automation goods are more substitutable in consumption, and when traditional capital is complementary to labor. These papers do not produce a Singularity, even with the rise of the robots.

There are three potential routes by which economic Singularity can arise. (I am grateful to Ben Jones for clarifying the distinction here.) The first is through accelerating technological change coming from technologies devised by superintelligent non-human agents, that is, when computers master stage three in machine intelligence. This is called the "superintelligent technology mechanism." I discuss this briefly in the next section but dismiss it as too speculative. A second mechanism is on the consumption side and would occur through a benign version of Baumol's cost disease called "Baumol's growth euphoria," discussed in the following two sections. A third and more subtle route is through increasing displacement of labor by capital and increasingly rapid capital deepening. This is called the "capital deepening mechanism." This will be the focus of the paper.

III. Rapid Technological Change through Superintelligent Innovation

A first possible source of extremely rising economic growth comes from rapid improvements in technology generated by superintelligent agents. This approach can be seen easily using a Cobb-Douglas production function of the form $Y_t = K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha}$. Here and below, assume that Y is output, K is capital, L is labor, A is labor-augmenting technology, s is the savings rate, and t is time. For most of the discussion, I assume the savings rate is constant. For a given rate of labor-augmenting technological change of h, the growth of output will be $g \rightarrow n + h$. Singularity quite naturally arises if technological change becomes extremely rapid.

The potential for accelerating economic growth has arisen occasionally as a curiosity in the literature on endogenous technological change. The key feature of the endogenous technology models is that knowledge is a produced input. One formulation would be that knowledge growth is proportional to the inputs into the production process. Here A_t is technological knowledge, Y_t is output, a fraction λ of output is devoted to inventive inputs, dA_t/dt is knowledge growth, and its growth is a function of inventive inputs, as in $dA_t/dt = \phi\left(\lambda Y_t\right)^{\beta}$. To simplify this greatly, assume that output is produced with labor, and that labor grows at a constant growth rate n. This implies that $\dot{A}_t/A_t = \phi(\lambda A_t L_t)^{\beta}/A_t = \phi\left(\lambda L_t\right)^{\beta} A_t^{\beta-1}$. If $\beta \geq 1$, which corresponds to increasing returns to inventive inputs, then the growth rate of output tends to infinity (see, particularly, Romer 1986, 1990).

The prospect of unbounded technological growth rates has not been taken seriously in the empirical growth literature for both technical and empirical reasons. The empirical reasons are that productivity growth has not accelerated in recent years. The technical reason is that it has unattractive assumptions about the knowledge-generation function, particularly increasing returns to inventive inputs. For useful discussions of the shortcomings of the model, see Jones (1995a, b).

How might technology accelerate? This might arise as an example of phase three of the "tasks of intelligent machines" discussed in the last section. A plausible mechanism comes where superintelligent machines begin writing "technological code" for most production processes. Note this is way beyond stage two's computer-assisted design or manufacturing because it involves intelligent or superintelligent computers designing new production functions. In reality, the evolution of computers designing new production processes is a giant leap from automation, which involves computers merely executing production functions (which is remote in many routine tasks today). Because this mechanism seems so far removed, it will be put aside.

IV. The Baumol Effect and Demand-Side Growth Euphoria

I begin by describing the forces from the demand side that might lead to rapid growth. This result is the mirror image of Baumol's cost disease and will be called *Baumol's growth euphoria*. Baumol and his coauthors emphasized the potential for low-productivity-growth industries to have rising costs, and potentially to slow aggregate economic growth (see Baumol and Bowen 1965; Baumol 1967; Baumol, Blackman, and Wolff 1985). However, depending upon the substitution properties, the impact might be to raise rather than lower aggregate productivity growth.

To begin with, sectors with relatively rapid productivity growth have relatively rapid price declines and will therefore generally experience a rise in relative consumption levels. The key question for the growth in aggregate consumption is whether those sectors with relatively rapid productivity growth have rising or falling shares in nominal expenditures. If low-productivity growth sectors dominate, that will produce stagnation; if high-productivity growth sectors dominate, that will produce rapid aggregate growth.

Baumol and his coauthors appeared to hold that the trend pointed toward stagnation because of rising expenditure shares of low-productivity growth sectors. For example, Baumol, Blackman, and Wolff (1985, 815–16) concluded as follows:

> The [real] output shares of the progressive and stagnant sectors have remained fairly constant in the postwar period, so that with rising relative prices, the share of total expenditures on the (stagnant) services and their share of the labor force has risen dramatically. …

Unfortunately, their analysis was made with old-style (Laspeyres) output indexes, so the calculations using real output shares were biased.

We can use a two-sector example to understand the forces at work. Assume that the economy has two sectors—call them information and handicrafts—produced by a single composite input. Output in each sector is linear in the composite input with divergent productivity trends. The rates of productivity growth are very high and very low, respectively. According to the Baumol mechanism, relative prices will be changing rapidly in favor of information.

If demand substitution is "inelastic" (technically, if the elasticity of substitution in demand between to two goods is less than one), then handicrafts eventually dominate expenditures, and the rate of growth of consumption will approach the rate of growth of productivity in the handicrafts sector. By contrast, if substitution is "elastic" (the elasticity of substitution in demand between to two goods is greater than one), then information dominates consumption, and the growth in consumption tends to the growth rate in the information sector. So here the critical parameter is the elasticity of substitution in the demand between the two kinds of goods. However, unlike supply-side effects discussed below, high demand-side substitutability does not lead to a Singularity.

A more rigorous statement is as follows for the two-sector example. Assume that there are two consumption goods (C_1 and C_2) that are information and handicrafts, respectively. Outputs are competitively produced with a single exogenously growing composite input, L. Productivity growth is assumed constant in each industry (at rates h_1 and h_2). Preferences are homothetic with a constant elasticity of substitution between the two goods, σ. Given these assumptions, prices in the two sectors are falling at rate h_1 and h_2 relative to wages. Total consumption as measured by an ideal index (such as the Törnqvist index) will be growing at rate $\beta_1 h_1 + \beta_2 h_2$, where β_1 and β_2 are the relative expenditure shares of the two goods. With some work, we can show that the ratio of the shares of the two industries is changing at the logarithmic rate of $(h_1 - h_2)(\sigma - 1)$.

So, for example, if $\sigma = 1.25$, $h_1 = 10$ percent per year, and $h_2 = 0$ percent per year, then the share of information will be rising at approximately $10\% \times 0.25 = 2.5\%$ per year (percent, not percentage points). Alternatively, to take a specific example of computers (formally, Information processing equipment), the relative price decrease over the last decade has been about 10 percent per year relative to other consumption. The share of computers in 2000 was approximately 2.0 percent. If the elasticity of substitution between computers and other goods was 1.25, then the share would grow to 2.6 percent after a decade. This is almost exactly the actual pattern over this period.

We can also easily calculate the Baumol effect for the two-sector example. The growth in consumption (in the superlatively measured Törnqvist index) equals the weighted growth of consumption, $\beta_1(t) h_1(t) + \beta_2(t) h_2(t)$. Under the assumptions in the last paragraph, the growth in the index of consumption over the decade would increase from 1.20 percent to 1.26 percent per year, or an increase of 0.006 percent per year. This is equal to the change in shares times the difference in the growth rates (change in shares $=$ 0.06 percentage points per year $\times$ growth rate difference of 10 percent per year). Note that with elastic substitution the growth rate in this model tends toward the growth in the high-productivity growth industry. The share of computers tends to one, so the weighted growth rate tends toward 10 percent per year in the simple example.

If we move to a multisector example, the analysis is analogous but more complicated. The analysis is laid out in Nordhaus (2008) and will be summarized here. Assume the growth rate of the ideal index of consumption is given by an almost ideal demand system in which consumption growth in each sector is a function of the growth in relative prices of the good and an income effect. If we assume that the income elasticities are uncorrelated with the changes in relative prices, then the average change in shares for each good will be determined by the average change in the relative price of that good times the price elasticity of demand minus one times the relative price movement. So, this is the analog of the two-sector example where the price elasticity replaces the elasticity of substitution. The aggregate effect is then the weighted average of this term plus errors due to exogenous growth rates plus income effects.

V. Empirical Tests of Baumol's Growth Euphoria

We can test for the Baumol or demand-side growth euphoria by looking at the relationship between the shares of different goods in total consumption and the trends in relative prices.

In a prior study of trends of major industries for the United States, I determined that there was a tendency for industries with falling relative productivity and rising relative prices to have rising shares of nominal output and of employment. This was consistent with the trend of the cost disease identified by Baumol and his colleagues cited above. I concluded, "There is a negative association of productivity growth with the growth in nominal output. In other words, stagnant industries tend to take a rising share of nominal output; however, the relationship is only marginally statistically significant."

An alternative approach for the present study focuses on consumption as a more natural place to examine substitution patterns. We can test the impact of the composition by examining whether those sectors that have the most rapid decline in prices tend to have rising or declining shares in expenditures. The identifying assumption is that prices move inversely with technological change, that technological change is exogenous, and that the errors in the demand relations are independent of technological shocks. Strictly speaking, it would be sufficient for a single sector to have rapidly falling prices and take over the entire consumption bundle, but we examine the more limited task of examining the trends of all components.

The US Bureau of Economic Analysis (BEA) has developed long-term data on consumption expenditures and prices starting in 1929. These data include 89 distinct sectors ranging in size from owner-occupied housing to food provided on the farm. In our analysis, we take a simple regression of the log of expenditure change on the log of price change for different periods. The results are shown in Table 1, which looks at both subperiods and the total period over the 1929–2012 record.

The coefficients differ by sector and period. The general pattern is for positive coefficients, indicating inelasticity of substitution for demand. If we examine the entire period from 1929 to 2012 or pooled subperiods of the total period, there is a

Table 1—Coefficient of log Price in Equation for Personal Consumption Expenditures

Period	Coefficient	t-statistic	Observations	p-value
1929–1948	0.25	1.10	48	0.012
1948–1969	0.90	2.59	54	0.012
1969–1990	0.06	0.37	83	0.714
1990–2013	−0.17	−1.58	90	0.118
1929–1969	0.15	0.50	48	0.617
1969–2012	−0.02	−0.26	83	0.796
1929–2012	0.44	2.04	48	0.047
Pooled, all subperiods	0.19	2.10	246	0.037

Notes: This table reports a regression of following:

$$\Delta \ln[expend_i(t)] = \alpha_0 + \alpha_1 \Delta \ln[price_i(t)/price(t)] + \varepsilon_i(t).$$

In this equation, $expend_i(t)$ is the share of personal consumption expenditures in sector i, $price_i(t)$ is the price index of sector i, and $price(t)$ is the aggregate price of consumption. Note that a positive coefficient indicates that a rising relative price increases the share of consumption expenditures and is therefore an indication of inelasticity of demand.

Source: US Bureau of Economic Analysis

Table 2—Share and Price Change for New Economy Sectors

	Change in prices (percent)	Change in share (percent)
Telecommunications	−2.9	−1.3
Video equipment	−11.1	−1.8
Information equipment	−21.1	4.7
Internet	−5.4	24.7
Telecommunications	−6.3	3.3
Photographic equipment	−3.2	−6.1

Notes: This table shows the average change in relative prices and in the shares of six information-technology sectors. The changes are for 1990–2012.

Source: Data are from the US Bureau of Economic Analysis

clear indication of inelasticity of substitution. These results are consistent with the analysis in Nordhaus (2008), which focuses on production patterns.

An alternative looks at personal consumption expenditures in major information technology sectors, shown in Table 2. These are defined as telecommunications, video services, information equipment, internet services, telephone, and photographic services. The prices of the new economy services have been declining steadily, but the trend in shares is mixed. Moreover, a statistical analysis of the six new economy sectors along the lines of Table 1 does not show a consistent pattern of elastic demand.

The size of the Baumol stagnation effect is small for the estimates that are provided here. We show in Figure 2 a calculation of the Baumol effect for selected well-measured industries.[1] The Baumol effect is the sum of the changes in shares

[1] These are selected both because the price indexes and real output measures are reliable and because they show a relatively large composition effect with large differences in output growth. The industries are food; imputed rental

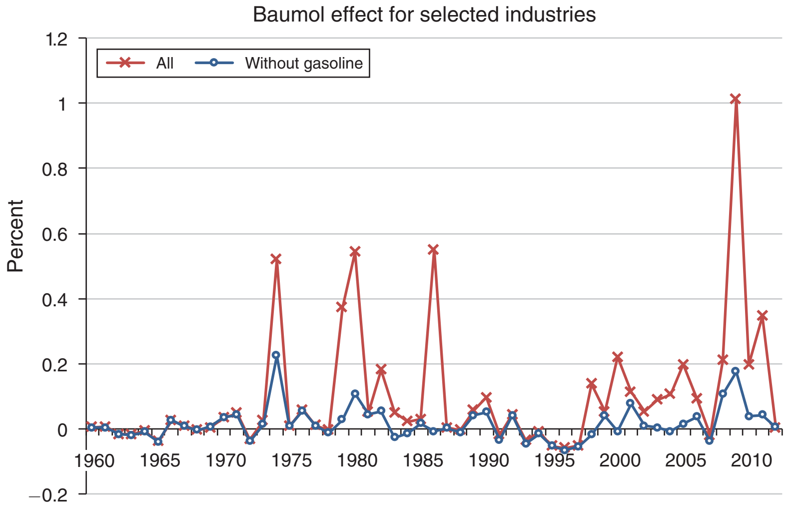

Figure 2. Baumol Effect for Selected Well-Measured Industries

Notes: This shows the net effect of changing shares on growth in consumption (measured as a Törnqvist index). A positive number in this graph indicates reduced overall growth; that is, industries with rising relative prices on average have rising shares of expenditures. Therefore, a positive number is a stagnationist force.

times the logarithmic price change. A positive number indicates a cost disease. For these industries, the Baumol effect is positive (subtracts 0.098 percent per year from aggregate consumption growth) if gasoline is included, and subtracts 0.015 percent per year without gasoline. In both cases, the effect is small, but in neither case is the effect to increase economic growth. The dominant effect of gasoline arises because it not only has a large share but is extremely price-inelastic in the short run.

These results indicate that the Baumol effect of changing shares in consumption is a force for stagnation rather than acceleration. In plain English, the sectors that are experiencing the most rapid price declines are also experiencing slight declines in expenditure shares. This tendency means that growth in aggregate consumption would slow over time if the underlying technological trends were stable. However, the impact of changing shares on the aggregate growth in consumption has historically been extremely small—in the order of minus 0.1 percent per year. The reason is that the shares of high- and low-productivity growth industries have not changed appreciably over the last two decades. So, this first test indicates no sign of rapid demand-side growth, or Baumol's growth euphoria. It is worth repeating, however, that even in the extreme case, this would not lead to a Singularity of the kind seen on the supply side.

of owner-occupied nonfarm housing; electricity; pharmaceutical products; new motor vehicles; motor vehicle fuels; telecommunication services; internet access; video and audio equipment; information processing equipment; magazines, newspapers, books, and stationery; and tobacco. They comprise about one-third of GDP in 2012.

VI. Supply-Side Singularity

The key accelerationist mechanism from the supply side operates through accelerating capital deepening. We can again start with a two-input model, similar to that of Simon above, to motivate the analysis. In this model, there are two factors of production and a single composite output that can be used for either consumption or investment. One input is either fixed or slowly growing, and it is usefully thought of as labor, or as a composite of labor and conventional slowly growing capital. The other input is assumed to be "information capital," which is produced by a rapidly improving technology.

In the simple two-input model, analogous to the Baumol effect, the key parameter is the elasticity of substitution in production. If the elasticity of substitution is greater than one, then the elasticity of output with respect to information capital increases (as does its income share), and the growth of productivity rises. If the elasticity of substitution is less than one, then information capital's elasticity (and income share) declines over time, and the growth of aggregate productivity tends toward the growth of the relatively fixed factor (labor or the slowly growing composite). In the unit-elastic Cobb-Douglas case, the output elasticities (and income shares) are constant, and productivity growth tends to a weighted average of the growth of the two inputs with constant weights.

There are other cases as well, such as multiple goods and multiple inputs, which are discussed below. However, the analysis is extremely simple in the one-good/two-input case. So, it seems best to start here and see what we find.

To develop the model further, we use a standard closed-economy neoclassical growth model with a constant savings rate and with a particular modification. Assume that labor is growing at a constant rate n and that all technological change is capital augmenting at a constant and rapid rate. In effect, we consider only information capital as an endogenous variable and sweep all other inputs into labor.

In the capital-deepening mechanism, the production function is of the following form:

$$Y_t = F(B_t K_t, L_t). \tag{1}$$

This specification assumes that technological change is purely capital augmenting, for simplicity at constant rate z. This leads to the following equation for the growth of output (g_t), where α_t is the elasticity of output with respect to capital, also assumed to be the income share of capital:

$$g_t = \alpha_t[z + sY_t/K_t - \delta] + (1 - \alpha_t)n. \tag{2}$$

For simplicity, assume that $z > n$, so there is capital deepening. Further, assume that the elasticity of substitution between capital and labor (EOSKL) is bounded above one ($\sigma > \overline{\sigma} > 1$). This leads to unbounded growth of output as the share of capital goes to unity:

$$g_t \rightarrow z + sY_t/K_t - \delta \rightarrow z + s(Y_T/K_T)e^{z(t-T)} - \delta \rightarrow \infty. \tag{3}$$

The algebra in equation (3) is complicated but can be simplified as follows. Assume that capital lives one period and is equal to investment in the prior period. In the limit as $\alpha_t \rightarrow 1$, this implies that $Y_t = B_t K_t = B_t s Y_{t-1} = B_0 e^{zt} s Y_{t-1}$, so $Y_t / Y_{t-1} = 1 + g_t = B_0 e^{zt} s \rightarrow \infty$. That is, the growth of output tends to infinity. Readers will recognize the results here as similar to the "AK" model of endogenous growth. In the simplest AK model, the elasticity of output with respect to capital is assumed to be one. The difference is that the AK model assumes this property, while the accelerationist model shows that this is a limiting result under the conditions of rapid growth in productivity of informational capital along with elastic substitution.

The surprise here is that the growth of output is unbounded. In effect, the economy is just information produced by information capital, which is produced by information, which in turn is producing information ever faster every year. We do not need to push this result to the absurd limit. Rather the three key points are (i) the value share of information capital in the input bundle is tending toward unity, (ii) as a result the contribution of information capital is rising, and finally (iii) because information capital is a produced input, the growth rate of output is accelerating.

Begin with a numerical example. The finding of unbounded growth is so surprising that we can perform numerical analyses to make sure it is not a mistake or simply a possibility for distant millennia. To get a flavor for the dynamics, perform a simple simulation. Assume that labor is constant, that all technological change is capital augmenting at 10 percent per year, and that the elasticity of substitution between labor and information capital is 1.25. Figure 3 shows a typical simulation of the income share of capital and the growth rates of output and wages. Growth goes off the charts after about 70 years. The result is the same as long as effective capital grows faster (or slower) than labor, but with slower (or faster) capital growth the time to the Singularity is lengthened (or shortened).

A second surprising result concerns the impact of rapidly growing growth on wages. Wages grow increasingly rapidly in this specification: wage growth reaches 200 percent per year in year 80. Capital eventually gets virtually all the cake, but the crumbs left for labor—which are really small pieces of the increasingly huge mountains of cake—are still growing at a phenomenal rate. The exact timing depends upon the parameters, but with elastic production and rapid capital productivity, the pattern always looks like Figure 3.

Turn next to model variants. The growth model described here is highly simplified, and we would be concerned about how sensitive the results are to alternative specifications. One clear simplification is the use of a constant savings rate. Instead, we might consider the outcome if savings responds to the prospect of rapid future technological change. Assume that the economy saves according to a Ramsey-Koopmans-Cass model with a constant elasticity of the marginal utility of consumption (EMUC) of θ. The optimal savings rate will depend upon the value of EMUC. If society behaves in a strict Rawlsean manner (where θ is indefinitely large), then the optimal savings rate will be low. In the case of a zero gross savings rate, the growth of output will then depend upon whether the rate of capital-augmenting technological change is larger or smaller than the depreciation rate on capital. With a smaller EMUC, such as logarithmic utility, the savings rate will tend to rise toward one, postponing consumption because saving

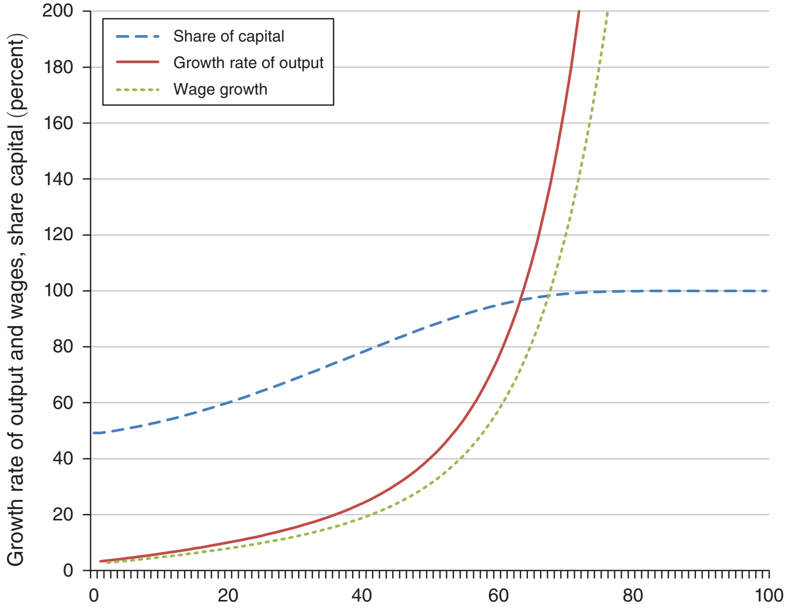

Figure 3. Simulation of a Growth Model with Rapid Technological Change in Capital and Elastic Substitution between Labor and Capital

is so productive of future consumption, and growth will be even higher than in the constant-saving case. Therefore, only in the case of extremely high (inelastic) EMUC and slow technological change will the basic result on accelerating growth be shut down by low endogenous saving.

VII. Tests for Supply-Side Singularity

Are we heading for the Singularity? If so, how far off is the rendezvous? Optimists believe that superintelligence could be achieved in a few decades based on the progress in computing power. We can apply the economic models developed above to examine observable economic variables that can distinguish supply-side accelerationism from stagnation or steady growth. We examine the following six tests:

1. Elasticity of substitution between capital and labor greater than one

2. Rising productivity growth

3. Rising share of capital

4. Accelerating growth in capital-output ratio

5. Rising share of information capital

6. Rising productivity growth hidden because of mismeasurement

Test 1: Elasticity of Substitution between Capital and Labor Greater than One.—We begin with evidence on the elasticity of substitution between capital and labor (EOSKL). Appendix A reviews recent data and the evidence from earlier studies. A summary is that the aggregate EOSKL is in the neighborhood of but cannot be reliably distinguished from 1. There is no reason why it should be constant over time, and it appears to be higher in recent years than in earlier years. So, on this critical parameter, the data speak softly if at all.

Appendix A also examines the elasticity of substitution between other inputs and information capital (EOSIK). The evidence here suggests a substitution elasticity of information capital for other inputs that is greater than 1. However, the most recent evidence indicates that the elasticity of substitution for the equipment component of information capital is less than unity for the period 2000–2015.

Test 2: Rising Productivity Growth.—The most important implication of the accelerationist growth model is that productivity growth is rising. This will show up as either rising labor productivity (LP) growth or rising total factor productivity (TFP) growth. While this is a central prediction, it does not provide a strong differential diagnosis because the rising productivity growth could come from other sources.

Figure 4 shows an estimate of total factor productivity for the United States for the period 1890–2018 by decade using two sources (Robert Gordon and the US Bureau of Labor Statistics). Productivity growth rate looks like an inverted U. It rose from the late nineteenth century and peaked in the 1950s, but has slowed to a crawl since 1970. For the latest period, 2000–2018, multifactor productivity growth grew at an average annual rate of only 0.3 percent per year. Figure 3 shows dramatically that there is no sign of any acceleration of multifactor productivity as of the most recent data for the United States. Even with the potential biases discussed below, this test is decidedly negative.

Test 3: Rising Share of Capital.—A central diagnostic forecast of Singularity is a rising share of capital in national income. (Note that the "share" in the growth model is the elasticity of output with respect to capital. The elasticity is not observable, so we use the income share, which would equal the elasticity under competitive conditions.)

Figure 5 shows the trend in the income share of capital (strictly speaking, all income other than labor compensation) over the period 1948–2018. One sectoral concept is the entire economy, while the other is the nonfarm business sector. The latter is better measured and provides a cleaner definition of capital income than the former, which includes a large component in owner-occupied housing as well as government capital. Note that capital income in the data includes many elements other than the net return to capital, such as depreciation, royalties on minerals, interest income, income of proprietors, production taxes, and some labor income classified as profits. Some analysts suspect that a substantial part of the increase in capital's share is either mismeasurement or is due to housing, so the estimates here are probably an upper bound on the share change (Elsby, Hobijn, and Şahin 2013 and Rognlie 2015).

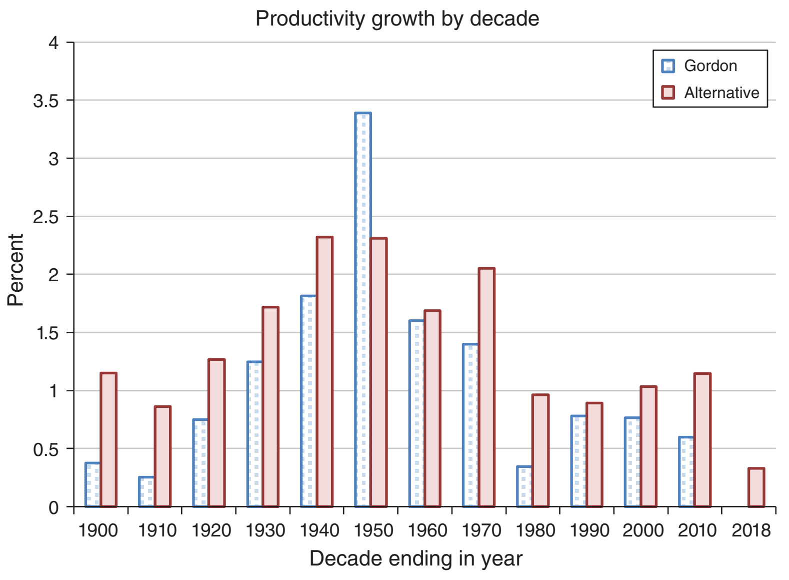

Figure 4. Total Factor Productivity Growth, 1890–2018

Notes: Alternative measure of productivity growth from Robert Gordon and as compiled by the author. For the period 1948–2018, the source is the BLS estimate of multifactor productivity.

Source: Nordhaus (2016), updated

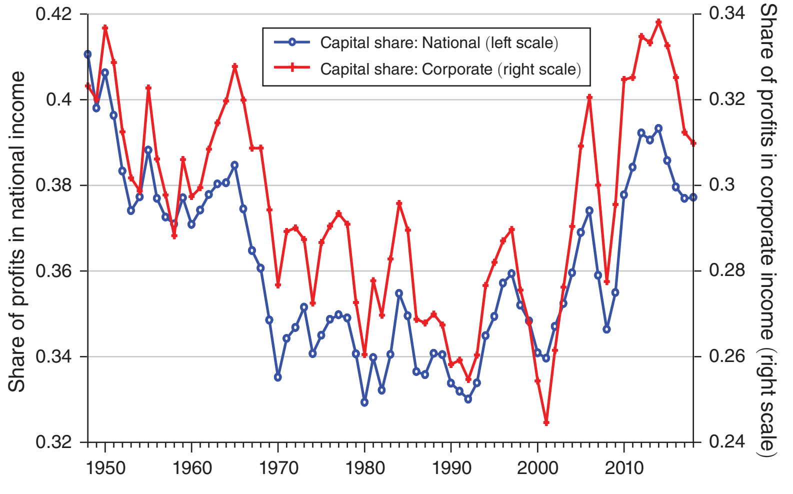

Figure 5. The Trend in the Share of Capital for the United States in the Overall Economy and Non-Farm Business Sector, 1948–2015

Sources: US Bureau of Economic Analysis and US Bureau of Labor Statistics

Table 3 shows regressions of the shares in the two sectors with and without breaks in trend in 1990 and 2000. Both show a small upward trend of about 0.4 percentage points per year since 1990 and 0.5 percentage points per year since 2000. This trend

Table 3—Regression Coefficients for Equation with Share of Capital as Dependent Variable and Time and Breaks in the trends in 1990 and 2000

	Trend	Trend since 1990	Trend since 2000
Non-farm business			
Coefficient	−0.02%		
t-statistic	−1.13		
Coefficient	−0.15%	0.36%	
t-statistic	−9.80	14.77	
Coefficient	−0.11%		0.45%
t-statistic	−9.07		10.30
Overall economy			
Coefficient	−0.02%		
t-statistic	1.97		
Coefficient	−0.15%	0.38%	
t-statistic	−14.52	10.56	
Coefficient	−0.10%		0.54%
t-statistic	−9.48		10.99

Notes: Number under coefficient is *t*-statistic. Share of capital is all income except compensation.

Source: Data from sources in Figure 5

is supportive of the accelerationist hypothesis at the raw data level. However, since we have a poor understanding of the reasons for the rise in capital's share, further research would be necessary to determine whether there is a link between this rise and a rapid rise in capital productivity (particularly in information capital).

Projecting future trends such as those of capital's share in Figure 5 is a primitive exercise. However, projections are useful to give some perspective on when the Singularity might become more apparent. Our simulation model shown in Figure 3 indicates that the acceleration in output is strong (with the growth rate crossing the 20 percent per year threshold) when capital's share crosses the 80 percent level. At the rate of increase from a forecast using the regression model underlying the last set of estimates in Table 3 (+0.54 percent per year), the 80 percent rate will not be reached until well after 2100. So, while the test is positive, Singularity is more than a century in the future using this diagnostic test.

Test 4: Accelerating Growth in Capital-Output Ratio.—Another important diagnostic concerns the real capital-output ratio. As is seen in the growth model sketched above, the capital stock (in efficiency units) will rise increasingly rapidly relative to output. The rise will come because informational capital grows rapidly, and also because informational capital takes a larger share of the capital stock.

For this test, we can look at the trends in the real capital-output ratio since 1960 (see Table 4). The capital stocks shown are different components of private capital corrected for quality by the Bureau of Economic Analysis and other government agencies. The output measure is gross business product. The first line shows that the overall capital-output ratio has been falling slowly over this period, although it has been close to constant in the last subperiod. Looking at the information capital components, these have been rising relative to output, but only at modest rates. In any case, the overall contribution of informational capital has been too small to lead to a rising capital-output ratio.

Table 4—Growth Rates of the Real Capital-Output Ratio, Different Sectors

Sector	1960–1990 (percent)	1990–2000 (percent)	2000–2012 (percent)
Private fixed assets	−0.5	−1.2	−0.2
Equipment	0.7	0.4	0.6
Nonresidential equipment	0.6	0.4	0.6
Information processing equipment	6.4	5.5	3.9
Computers and peripheral equipment	na	21.1	6.3
Intellectual property products	4.3	4.4	4.1
Nonresidential intellectual property products	2.0	2.1	1.8
Software	18.2	10.2	3.2
Research and development	2.5	0.4	1.4

Source: Bureau of Economic Analysis

Test 5: Rising Share of Information Capital.—A further test is that informational capital should be a rising share of the capital stock. Indeed, as the economy approaches the Singularity, the share of informational capital should approach 100 percent.

Table 5 shows the shares of informational capital in total private assets. (These are the current-cost net stock of private fixed assets from the BEA.) It is clear that informational capital is becoming a more important part of the capital stock. The growth is particularly strong in intellectual property products. Software has grown sharply, while computers and information processing equipment has stagnated. So, this test would appear to conform to the Singularity view, although there is still a long way to go before these sectors dominate investment.

To determine whether an inflection point is in the near future, we project the share of informational capital into the future at the growth rate for the 1960–2018 period. Our numerical example suggests that the growth rate begins to accelerate when the capital share exceeds 80 percent of income. Our extrapolation of Table 5 indicates that this would not occur within the next century, so the Singularity appears at best distant by this test.

Test 6: Rising Productivity Growth Hidden because of Mismeasurement.—A major question muddies the analysis, however. Are measurement errors hiding rapid productivity growth? One concern with the tests above is that productivity growth is underestimated. Are measurement errors hiding rapid productivity growth? Hal Varian, the chief economist at Google, argues that there is an explosion of productivity underway because of the devices, apps, and other digital innovations coming out of Silicon Valley. "There is a lack of appreciation for what's happening in Silicon Valley because we don't have a good way to measure it" (Aeppel 2015).

The modern analysis of computers and productivity often dates from Robert Solow's famous 1987 remark, "You can see the computer age everywhere but in the productivity statistics." The tests above suggest that his remark seems to hold almost equally well three decades later. The issues involved in measuring the contribution of new and improved goods and services have been carefully studied and raise several thorny issues (see Gordon 2016 for an extensive discussion). The most important shortcomings arise from the improper measurement of the prices for goods that

TABLE 5—SHARE OF INFORMATION CAPITAL IN TOTAL CAPITAL

Sector	1960 (percent)	1990 (percent)	2017 (percent)
Equipment	17.57	19.18	14.88
Nonresidential equipment	17.34	18.93	14.75
Information processing equipment	1.77	4.48	3.38
Computers and peripheral equipment	0.02	0.70	0.56
Intellectual property products	2.80	4.85	7.11
Nonresidential intellectual property products	2.80	4.85	7.11
Software	0.01	0.64	1.42
Research and development	1.52	3.01	4.57

Source: Bureau of Economic Analysis

are either new or show rapid quality improvement. Recall that "real output" growth is nominal output growth less the rate of change of the price of the good. So, if price increases are overstated, as is the case with insufficient quality adjustment, then real output increases will be understated.

We can illustrate the issue for the sector I will call "new economy communication." This poses some of the most difficult issues in price measurement, both as a new product and with quality change. There are three components here of consumer expenditure: telephone and facsimile equipment; telecommunication services; and internet access. These totaled 2.3 percent of consumer expenditures in 2015. The major item involving quality change is cell phone equipment.

It is difficult to measure the improvement in quality because of the rapid improvements in cell phone design along with the many bundled applications. Until 2018, the BLS did not make explicit quality adjustments for cell phones, but at that time it began to adjust for quality using hedonic methods.[2]

It is interesting to take one example of cell phone changes to illustrate how productivity estimates may be biased: the camera. In an iPhone 6, the camera is estimated to cost about \$20. Recent reviews suggest that it performs as well as a point-and-shoot camera (costing perhaps \$100–\$200) but less well than a digital (costing \$1,000+). So, to a first approximation, the cost of a camera declined by a factor of about 10. The marginal cost of a photo is essentially the time-cost. Hal Varian estimates that the worldwide production of photos has increased 20-fold over the last decade or so. A crude analysis would suggest that the economic surplus from this quantitative expansion would be about \$25 billion per year in 2019. This is approximately 10 percent of new economy communication consumption, so not a trivial amount.

The camera example just scratches the surface for cell phones. Other factors are convenience, safety, locational services, messaging, and the benefits of footloose connectivity. A recent study by Byrne, Sichel, and Aizcorbe (2019) provided a hedonic estimate of the price of smartphones, but looking primarily at their components and not at other factors just mentioned. Their preferred index, with coefficients on characteristics that change over time, falls at an annual average rate of 16 percent

[2] This comes from a personal communication with BLS staff as well as a description of pricing on the BLS website.

from 2010 to 2018. The BLS does not provide its index of smartphones, so we do not know the magnitude and significance of mismeasurement of cell-phone output.

While the mismeasurement in some areas is difficult to calculate, as we saw with cell phones, we can quickly dispose of one part of the issue, which involves the use of IT by companies. To the extent that IT *inputs* are incorrectly measured, those would show up as productivity for the industry. If, for example, underpriced internet services vastly increased the ability of airlines to utilize their fleet more efficiently, then measured productivity growth of airlines would rise. Alternatively, if better IT allows companies to reduce the defect rate in production, then again, productivity would rise. So, the mismeasured IT going as intermediate products or capital services to business would not lead to underestimated aggregate productivity growth.

What are the proportions of consumer versus business in information technology? We can look at detailed input-output tables to get an idea of the magnitudes. Taking the major 11 IT sectors,[3] we can divide gross output into that part going to consumers and that going to businesses. The former is included as personal consumption expenditures, while the latter are investment or intermediate purchases. Looking at the input-output structure for 2002, there were $1,217 billion in domestically purchased IT goods and services (about 11 percent of GDP). Of these, 77 percent were purchased by businesses, 23 percent were by consumers. The major consumer service was telecommunications, where consumers purchased about half of total output. Given these numbers, it seems likely that most of the productivity impacts of IT will be captured in either business output or business productivity.

The IT purchases by consumers comprised 1.4 percent of GDP over the last two decades. If productivity growth for these products were underestimated by 10 percent per year (surely an upper bound on the number), aggregate productivity growth would be underestimated by 0.14 percent per year. Even this extreme assumption would do little to change the shape of the productivity slowdown over the last few decades.

The final test of Singularity will look at circumstantial evidence in productivity estimates *outside* information-technology industries. Here is the idea: Suppose that there is a rapid growth of productivity in IT but not outside those industries, and all outputs and inputs are correctly measured. Further, suppose the standard assumptions behind calculations of multifactor productivity (MFP) are correct. We would under these assumptions see rapid growth in MFP in the IT industries alongside rapid growth in the IT *inputs* into other industries, but normal MFP growth outside IT. For example, suppose that IT inputs to airlines are growing at a rapid rate and are a major share of inputs; in this case, inputs and outputs of airlines will be growing rapidly, but airline MFP will be creeping along at a normal pace.

By contrast, assume that government statisticians are greatly underestimating the rate of improvement of IT capital and other inputs. This will lead to an underestimate of the inputs to airlines, *and a large increase in measured MFP improvements*

[3] The sectors were computer and peripheral equipment; audio, video, and communications equipment; semiconductors and electronic components; electronic instruments, software publishers, cable networks and program distribution, internet publishing and broadcasting, telecommunications, data processing services, other information services, and computer systems design and related services. Data are from www.bea.gov.

in airlines. This reasoning suggests that mismeasurement of IT inputs would show up as an acceleration of MFP in industries outside the IT sector. Of course, this kind of test would show up most powerfully where the output of the outside industry is well-measured (perhaps steel and corn, but not so much in financial and health services).

We can test for the presence of major IT mismeasurement by examining MFP measures constructed by the Bureau of Labor Statistics for detailed industry groups, which are available for the period 1987–2014. Since the major IT improvements have taken place since 2000, we would see major mismeasurement as a sharp increase in measured MFP growth since that time. I have taken the most recent data and sorted it for MFP growth 1987–2014, concentrating on well-measured industries. Table 6 shows average annual MFP growth for 1987–2014, for two subperiods, and the acceleration from the first to the second subperiod.

There is only one not-surprising standout industry, computer and electronics products, with average MFP growth of 8 percent per year over the entire period. But this industry showed a sharp deceleration from the first to second subperiod. A few other industries show solid but hardly revolutionary changes. There is no general acceleration in MFP growth, nor is there any obvious pattern of acceleration among industries. The industries with the largest acceleration are data processing, internet publishing, and other information services; oil and gas extraction; motion picture and sounds recording industries; computer systems design and related services; funds, trusts, and other financial vehicles; and forestry, fishing, and related activities. Some are IT intensive, while others are not.

To summarize: If the inputs of information technologies were improving rapidly but were not captured in the statistics, this would show up as rapid growth (and perhaps acceleration) of MFP. It simply is not happening in a broad-based fashion. So, this test is also negative on Singularity.

VIII. Summary of Tests for Singularity

Table 7 shows a summary of the six tests of Singularity. Four of the six tests are negative or ambiguous for Singularity, while two are weakly positive. We can also calculate for the two positive tests how far we are from the point of Singularity. I define Singularity as a time when the economic growth rate crosses 20 percent per year. Using simple extrapolation for the two positive tests, the time at which the economy might plausibly cross the Singularity is beyond 2100.

IX. Interpretations and Elaborations

The theory and tests proposed above raise several issues of interpretation. I consider some important ones in this section.

A. *Old Wine in New Bottles?*

One reaction to the economics of the Singularity is that it is just a continuation of past trends. An early reader of this paper commented that machines can fly and

Table 6—Average Annual Growth in Multifactor Productivity, 1987–2014 (*percentages*)

Sector or industry title	MFP growth (annual average) 1987–2000	2000–2014	1987–2014	Productivity acceleration
Computer and electronic products	11.0	4.5	7.6	−6.5
Air transportation	2.1	3.2	2.7	1.1
Support activities for mining	2.8	2.3	2.6	−0.5
Securities, commodity contracts, and investments	4.6	0.3	2.4	−4.3
Warehousing and storage	2.4	1.5	1.9	−0.9
Water transportation	0.9	2.5	1.8	1.6
Crop and animal production	1.8	1.0	1.4	−0.8
Broadcasting and telecommunications	0.0	2.6	1.3	2.6
Rail transportation	2.1	0.6	1.3	−1.5
Publishing industries, except internet [includes software]	0.7	1.8	1.3	1.1
Pipeline transportation	1.3	1.1	1.2	−0.2
Petroleum and coal products	1.6	0.5	1.0	−1.1
Oil and gas extraction	−1.6	3.5	1.0	5.1
Information	−0.4	2.3	1.0	2.7
Accommodation	1.1	0.8	1.0	−0.3
Printing and related support activities	0.0	1.6	0.9	1.6
Textile mills and textile product mills	1.0	0.5	0.8	−0.5
Utilities	0.4	0.9	0.7	0.5
Motion picture and sounds recording industries	−1.3	2.2	0.5	3.5
Plastics and rubber products	0.7	0.1	0.4	−0.6
Truck transportation	1.1	−0.2	0.4	−1.3
Transportation equipment	−0.3	0.8	0.2	1.1
Primary metals	0.4	0.0	0.2	−0.4
Nonmetallic mineral products	0.4	−0.2	0.1	−0.7
Food services and drinking places	0.1	−0.1	0.0	−0.2
Wood products	−0.5	0.2	−0.1	0.8
Fabricated metal products	0.1	−0.4	−0.2	−0.5
Furniture and related products	0.1	−0.4	−0.2	−0.5
Paper products	−0.1	−0.4	−0.3	−0.2
Transit and ground passenger transportation	0.5	−0.9	−0.3	−1.4
Food and beverage and tobacco products	−0.5	−0.2	−0.3	0.3
Chemical products	−0.8	−0.7	−0.8	0.1
Data processing, internet publishing, and other info. services	−5.0	3.0	−0.9	8.0
Apparel and leather and applied products	1.2	−3.8	−1.5	−5.0

Notes: The table shows the rank of MFP growth over the entire period (third numerical column). The key finding is that MFP growth is limited to computers. The last column indicates that there is no general pattern of accelerating MFP over the subperiods.

Table 7—Results of the Singularity Tests and Time to Singularity

Source	Result of test	Time until singularity
Test 1: Elasticity of substitution between capital and labor greater than one	Amibiguous	x
Test 2: Rising productivity growth	Negative	x
Test 3: Rising share of capital	Positive	>100 years
Test 4: Accelerating growth in capital-output ratio	Negative	x
Test 5: Rising share of information capital	Positive	>100 years
Test 6. Rising productivity growth hidden because of mismeasurement	Negative	x

Source: Earlier figures and tables

computers can carry out the most complex calculations, but these feats have not made humans obsolete. Moreover, it takes years or decades for businesses to learn how to apply new technologies like steam and electricity to the economy. Big data can make big calculations, but they have not replaced teachers or judges, and have made virtually no inroads into carpentry.

While demurring to these observations, it is important to distinguish the theory of the Singularity from earlier innovations. Steam, electricity, and internet shopping—these are multipliers of human work. The nature of the Singularity is different: it is to replace the unique feature of human labor, which is its intelligence. Labor productivity doubles when the number of pilots in an airplane goes from two to one, but it goes up infinitely when the number of pilots goes from one to zero for drones.

Another way to think of the prospect of Singularity is as a race between task automation and task innovation. The scope of human tasks has two dimensions, lateral and temporal. The lateral dimension involves all the things humans do today, such as piloting or carpentry, while the temporal dimension reflects the fact that innovation expands the list of tasks, including new ones such as code-writing in C++ or designing autonomous cars.

Perhaps AI can eventually take on most of today's lateral list of tasks with decades of work and trillions of lines of code. However, tasks are evolving, so by the time AI has mastered the tasks of 2019, a whole new list of tasks have sprung up. This is the analysis developed in Acemoglu and Restrepo (2018). They have a framework in which tasks previously performed by labor are automated, while innovation creates more complex tasks in which labor has a comparative advantage.

While this picture of a race between task automation and task innovation helps understand the nature of the evolution of human work, it misses the central feature of Singularity. With the development of superintelligent machines, the task of automating tasks is taken over by task-automating machines. If these machines become truly superintelligent, the pace of task automation will surpass the pace of task innovation. All of this is of course highly speculative, but it helps show why the Singularity differs from earlier patterns of task evolution.

B. *Violations of Basic Physical Laws?*

An objection that might arise is whether the Singularity violates basic laws of nature. All processes need minimal energy, and energy is limited if superabundant. Other potential limiting resources are fresh and clean water, oxygen, and exotic minerals to build machines. Some would invoke the second law of thermodynamics, which holds that increasing order must be offset by increasing disorder elsewhere.

The issues here are too deep to be adequately treated in the present study. While some resources are indeed needed for all production processes, the inputs can in theory be reduced sharply, and potentially even more rapidly than production increases. This is vividly illustrated for computation. An early computer was the ENIAC (shown at the upper left in Figure 1). It required about 150 kW to operate, or approximately 55 watts per floating point operation (flop). A desktop computer today requires about 75 watts to produce 10^{13} flops. While this is only an approximation, this calculation indicates that the energy requirement for computation has declined by a factor of 10,000,000,000,000. In recent years, energy use has declined at approximately the rate of improvement of computers.

So, the bottom line on resources is that improvements in material use and miniaturization can overcome the physical limitations on accelerating growth. As Richard Feynman said, "There is plenty of room at the bottom."

C. *Heterogeneous Labor in the Growth Model*

The Simon-type growth model of information and productivity analyzed above has the shortcoming that it assumes homogeneous output, capital, and labor. Heterogeneous output is considered in the Baumol example. We consider in this section the interesting implications of adding heterogeneous labor to the analysis.

Economists have generally found that skilled workers are more adaptable to rapid changes in information technology than middle-skilled, manual, or unskilled workers. The process is summarized nicely by Autor (2014, 135):

> 'Routine tasks' [are ones] that follow an exhaustive set of rules and hence are readily amenable to computerization. Routine tasks are characteristic of many middle-skilled cognitive and manual activities, such as bookkeeping, clerical work and repetitive production tasks. Because the core tasks of these occupations follow precise, well-understood procedures, they are increasingly codified in computer software and performed by machines. This force has led to a substantial decline in employment in clerical, administrative support and, to a lesser degree, production and operative employment …

We can extend the Simon model to include heterogeneous labor by considering some polar cases. Assume as one example that unskilled labor is a perfect substitute for informational capital, while the other input is skilled labor. As above, skilled labor has high but imperfect substitutability with capital. We then directly apply the analysis above. The marginal product and wage of unskilled labor fall proportionally with capital prices. More realistically, if there is a reservation wage for unskilled labor, say because of income support, unskilled labor is essentially worthless and disappears from the labor market.

Is this an absurd result? As a historical analog, consider the fate of human adding machines of the nineteenth century. As explained in Nordhaus (2007), there was a revolution in the employment of human calculators around 1900. In his nineteenth century book on calculation, Orton writes, "To be able to add two, three or four columns of figures at once, is deemed by many to be a Herculean task, and only to be accomplished by the gifted few, or in other words, by mathematical prodigies" (Orton 1866, p. v). The prodigies who could add up columns of numbers rapidly, called "lightning calculators," were at a premium. The advent of mechanical and electronic calculators changed all that. Aside from quiz shows, there is no demand today for lightning calculators. Such would be the fate of unskilled labor in this simple two-labor model as we approached the Singularity.

What is the economic fate of skilled labor? In the simple two-labor-input model described here, skilled labor would have the same future as labor in the one-labor

Simon model. Its share in national income would tend to zero as capital took over the economy. But skilled labor would be fully employed, and its wages would begin to rise rapidly as shown in Figure 3. We would see social and economic polarization with a vengeance.

Perhaps the pattern of impacts would be reversed, as is suggested by Autor (2014). Perhaps the work of skilled labor would be substituted by information technology while unskilled labor would be the only group not susceptible to substitution by information technology. Perhaps, patients would be diagnosed and treated by computers rather than doctors. Classes would be taught online by computerized instructors and virtual teaching assistants rather than academic scribblers. Central banks would finally, following Milton Friedman's vision, be run by a computerized rule rather than imperfect discretion of bankers. Workers just hook up the monitors, plug in the machines, and make sure that the Fed has the latest operating system. Since the skill ladder is a two-way street, skilled workers would abandon their professional degrees as the skilled jobs disappear and all humanity becomes unskilled apprentices to computers. We are then back to the Simon model, but in this case, with the one factor being unskilled labor. Surprisingly, there is much greater labor-market equality than in the first example. So, with heterogeneous labor, the fate of different skill groups depends critically on their substitutability with information.

D. *The Euthanasia of the Laboring Classes*

As growth accelerates with superintelligent capital, the rate of return on capital and real interest rates fall to zero. This was an outcome envisioned by J. M. Keynes in a chapter from *The General Theory* (Keynes 1935):

> [There would be an] increase the stock of capital up to a point where its [marginal product] had fallen to a very low figure …. Now, [this] would mean the euthanasia of the rentier, and, consequently, the euthanasia of the cumulative oppressive power of the capitalist to exploit the scarcity-value of capital.

Keynes's analysis predated the pioneering work on production functions that clarified the key role of the elasticity of substitution on factor shares, and as a result, he saw only one of several possible outcomes. Keynes's scenario described a growth path in which the elasticity of substitution between labor and capital is less than one; accumulation in the inelastic case, therefore, drives not only the rate of return to zero but also the share of capital to zero.

However, the accelerationist case leads to the opposite outcome, where the share of capital goes to unity. In this outcome, we thus would see the euthanasia of the laboring classes in the sense that all of national income eventually goes to the owners of capital. Workers would be well-paid but would control a vanishing part of national output. However, as long as corporations own most of the capital, and people or human institutions (including governments through taxation) own corporations, capital income will indirectly flow through to humans. Since national income equals national output, average income will be growing increasingly rapidly.

How this will play out in terms of individual equality or inequality goes beyond economics to politics, tax and benefit systems, and the nature of dynastic savings. Will the incomes be captured by the Schumpeterian classes—the innovators who design machines and write software for them? Alternatively, by the wealthy who subvert institutions to increase their wealth? By those who are the last humans who are complements rather than substitutes for information, perhaps as gardeners or butlers? Perhaps by those who control the intelligent machines before they take over?

Fortunately, the euthanasia of the laboring classes is far off and will flash warning signals so that, if it does occur, humans will have time to contemplate the social structures of such an era.

E. *Autonomous Agents in Warfare*

Most people today are focusing on the growing threat of computer hacking—into elections, tax returns, and our phones. Less prominent but perhaps more ominous is the use of information technology in warfare. There are very powerful incentives to develop autonomous and robotic activities because of the winner-take-all nature of military technologies and because the dangers of combat make nations averse to risking lives.

The key word in the last paragraph is autonomous. The US Department of Defense defined these as "weapon systems that, once activated, can select and engage targets without further intervention by a human operator." This definition suggests the ability of such systems to assess the situational context on a battlefield and to decide on the required attack according to pre-programmed rules and battlefield information.

Some of the key developments in IT warfare are the following. Drone aircraft such as the Predator have the capability to identify targets and fire missiles. Daksh is a battery-operated, remote-controlled robot on wheels that can recover and defuse bombs. Guardium is a small Israeli tank-like surveillance vehicle that operates completely autonomously to guard the Gaza border. PackBots are a series of small robots used to identify bombs, collect air samples in hazardous sites, and sniff for explosives. SWORDS is a small American tank-like vehicle that is remote-controlled at this time. The Samsung SGR-A1 is a South Korean military robot sentry, armed with sensors and a machine gun that can operate autonomously and is designed to replace human counterparts in the demilitarized zone at the South and North Korea border. More advanced versions of these are under development. It is possible to envision that a rogue nation will develop genetically engineered super-humans to fight alongside robots.

While the automation of warfare is only in its infancy, we can examine the impact to date. The share of compensation in total output for US defense spending has risen slightly over the last two decades, so on that test the accelerationist hypothesis is not supported. Battle deaths in recent wars (in Iraq and Afghanistan) are down sharply from earlier wars (Vietnam and Korea), and this is undoubtedly in part due to better information and smart weapons. The success of cyberweapons in warfare (as far as we can tell from public sources) is limited to date, perhaps setting back Iran's nuclear program by a year or so, but there are many frightening scenarios.

The gruesome wars of the present (in Syria to take one example) rely largely on last-generation technology of dumb bombs, chemical weapons, artillery barrages, terror, and torture. So, the bottom line on the role of IT in military technologies is that it has not moved substantially toward replacing human labor up to now.

F. *Competition among the Superintelligent*

If superintelligent agents develop, we must contemplate the prospect of competition among rival powers. The parallel here is to the game-theoretic dynamics of weaponry. Even though the innovators (of bows and arrows, machine guns, tanks, nuclear weapons, and drones) had an initial advantage over their adversaries, their advantage was temporary. Even the most closely held technological secrets diffuse slowly around the world.

We must therefore assume that those who develop the engines of superintelligence will eventually find they are soon shadowed by their military, commercial, and political adversaries. Moreover, to the list of adversaries will be added the superintelligent machines themselves.

We might take comfort in Asimov's Three Laws of Robotics, of which the First Law is, "A robot may not injure a human being or, through inaction, allow a human being to come to harm." However, to take refuge here would surely be super-naïve. It would only take one unethical designer to launch a superintelligent agent who did not incorporate the Laws of Robotics. This would probably launch an arms race among rival superintelligent powers. So, the point here is that the approaching Singularity is not one of unambiguous economic and social improvement. This was appreciated by nuclear weapons developer John von Neumann (1955):

> Useful and harmful techniques lie everywhere so close together that it is never possible to separate the lions from the lambs. This is known to all who have so laboriously tried to separate secret, classified science or technology (military) *f*rom the open kind; success is never more nor intended to be more than transient, lasting perhaps half a decade. Similarly, a separation into useful and harmful subjects in any technological sphere would probably diffuse into nothing in a decade.

X. Concluding Comments on Singularity

So, the conclusion as of today is that "the Singularity is not near." This conclusion is based on several tests that place the theory of the Singularity within the context of economic growth theory. Much of the computer science literature on the Singularity examines the growth in specific sectors or processes (such as flops or storage), but the economic perspective insists that the growth must be weighted by the economic valuation of the good or service.

The central analytical insight about the Singularity is this: If information and conventional stuff (non-information inputs or outputs) are elastic substitutes either in consumption *or* in production, then growth will rise, perhaps extremely rapidly. Singularity, in the sense of unbounded growth, can only arise with elastic substitution on the supply side. If information and conventional stuff are inelastic in production *and* consumption, then rapid improvements in information technology will eventually be irrelevant to the economy.

The major insight of economics is to emphasize the heterogeneity of both inputs and outputs of the economic system. It is surely true that technological change in the production of raw computation has been phenomenal over the last century. But economic activity is more than bits. For increasing capabilities of computers to lead to the Singularity would require that AI could encompass all human activities, not just add numbers, solve equations, play chess, and interpret speech; but also lay hands on patients, babysit and comfort children, and mediate disputes.

Whereas computerized AI might do many routine tasks, the non-routine tasks are less easily programmed, and they evolve in response to the economic environment, including the environment of artificial intelligence itself. Particularly if we view the world with potential superintelligence as a competition between humans and machines, then we definitely would need a team of humans to consider how to protect humans from machines. So, one occupation at least would survive into the Era of Singularity.

Whether other sectors and tasks would be immune to the rise of superintelligence is an open question. The empirical question is the degree of substitutability between information and human efforts. Given the complexity of both humans and jobs, it is unlikely that the question can be decided a priori. The analysis above indicates that information and computers will come to dominate the economy only if the informational capital takes a rising share of inputs. This requires that the expenditure shares or input cost shares of information rise over time, which in turn requires that the volume of inputs rises more rapidly than the relative prices fall. We can call these the "substitution tests" to be concise.

There are six tests on the supply side. The conclusions from the empirical tests are that the substitution tests fail or are ambiguous for four of six tests and succeed barely for two of the six tests. However, the growth trajectories of the variables which pass the test (the share of capital in total income and the share of informational capital in total capital) are extremely slow. Projecting the trends of the last decade or more, it would be in the order of a century before these variables would reach the level associated with the growth Singularity. The conclusion is therefore that the economic Singularity is not near.

Appendix A. Estimates of the Elasticity of Substitution of Capital and Labor

As is clear from the modeling developed in the paper, a key issue on Singularity concerns substitution between information capital and other factors of production, or more generally the elasticity of substitution between capital and labor (EOSKL). A substantial body of work exists on this question. The basic conclusion is that it has been difficult to determine whether the EOSKL is greater than or less than unity.

Elasticity of Substitution of all Capital and Labor

The constant-elasticity-of-substitution (CES) production function was introduced by Arrow et al. (1961). They found substitution elasticities generally below 1 in their study. Berndt (1976) used alternative estimators and found that estimates clustering around 1.2. A survey of the literature by Chirinko (2008) found the weight of the evidence at that time for an EOSKL less than one. A more recent study by Karabarbounis and Neiman (2014) examines the more recent period and estimate the EOSKL at around 1.25. An approach aggregating micro data to macro estimates by Oberfield and Raval (2014) finds an EOSKL for US manufacturing of 0.7. It is hard to find any consistent answer on the EOSKL in the existing literature.

Since we are primarily interested in the aggregate data, it would be appropriate to examine the EOSKL for major sectors. For the United States, during most of the last century, the capital-labor ratio was rising. The share of labor in US national income rose gradually from 1929 to 1980, then fell gradually after that time. If technological change were unbiased, this would indicate that the EOSKL was less than 1 during the first period and greater than 1 during the second period. Karabarbounis and Neiman (2014) examine labor's share globally in both the corporate sector and the overall economy and found that it has declined about 5 percentage points from 1975 to 2012.

It will be useful to examine a simple aggregate production function to estimate the EOSKL. A high-quality dataset for the private business sector has been prepared by the BLS for multifactor productivity. Assume the production function takes the form where all technological change is factor augmenting, so we write the CES production function as $Y_t = [\beta_t (A_t K_t)^{-\rho} + (1 - \beta_t)(B_t L_t)^{-\rho}]^{-1/\rho} \varepsilon_t$. In this specification, Y_t is output, A_t and B_t are the levels of factor-augmenting technology, β_t is the bias of technological change, $\sigma = 1/(1+\rho)$ is the EOSKL, and ε_t is a random error term.

Assuming factors are paid their marginal products and that there is no bias to distribution ($\beta_t = \bar{\beta}$), this implies that the ratio of capital's share to labor's share is $\alpha_t/(1-\alpha_t) = \varphi(B_t/A_t)^{1/\sigma}(K_t/L_t)^{(\sigma-1)/\sigma}$, where α_t is the factor share of capital. For identification, I assume that inputs of labor and capital are exogenous.

Table A1 shows the estimates of the EOSKL for the entire period 1948–2014 and subperiods with and without a trend and with and without an AR correction. If we assume that technological change is Hicks-neutral, the trend term (B_t/A_t) is constant, and the estimate without a trend is appropriate. Estimating this equation over the period 1948–2014 without the AR1 correction yields an EOSKL of 1.08 for the entire period, with a value of 0.93 for the first part of the period (1948–1965), and a value of 1.19 for the second part of the period (1965–2014). The appropriate test is whether the coefficient is significantly different from 1, shown as a *t*-test in the last column. Most of the tests indicate that the EOSKL is not significantly different from 1 for most specifications.

However, if we assume that there is a persistent bias to technology, so $(B_t/A_t) = (1+h)^t$, we would include a trend term. As shown in the "Trend" rows of Table A1, the EOSKL is less than unity for all trend estimates. It is significantly less than 1 without an AR1 correction, but is not significantly different from 1 with an AR1 correction.

A summary is that the aggregate EOSKL is in the neighborhood of 1. There is no reason why it should be constant over time, and it appears to be higher in recent years than in earlier years. So, on this parameter that is so critical to understanding the likelihood of the Singularity, the historical evidence is ambiguous.

Table A1—Estimates of the Elasticity of Substitution of Capital and Labor for the US Non-Farm Business Sector, 1948–2014 and Subperiods

Trend?	AR1?	Period	Coefficient	Standard error coefficient	t-statistic (from 1)
No trend	No AR1	1948–2014	1.08	0.02	3.55
No trend	AR1	1948–2014	1.12	0.09	1.35
Trend	No AR1	1948–2014	0.66	0.09	−3.72
Trend	AR1	1948–2014	0.75	0.13	−1.90
No trend	No AR1	1948–1965	0.93	0.07	−1.06
No trend	AR1	1948–1965	0.97	0.15	−0.18
Trend	No AR1	1948–1965	0.48	0.13	−4.10
Trend	AR1	1948–1965	0.54	0.19	−2.40
No trend	No AR1	1965–2014	1.19	0.04	4.57
No trend	AR1	1965–2014	1.27	0.17	1.60
Trend	No AR1	1965–2014	0.83	0.16	−1.07
Trend	AR1	1965–2014	0.85	0.19	−0.78

Elasticity of Substitution of Information Capital and Other Inputs

While there is a vast literature on the elasticity of substitution between all capital and other inputs, there are few studies on the elasticity of substitution between information capital and other inputs (EOSIK). The major study of this question is Dewan and Min (1997). They find an EOSIK slightly above unity in a CES-translog production function. However, the estimate has very low precision and is not significantly different from 1.

We can use the simple specification shown above to estimate the EOSIK from sectoral data. The growth rate of the quantity of capital can be written as the weighted average of its components, where the weights are the current cost shares. If the EOSIK for individual components is above unity, then the shares of those components with prices falling relatively rapidly will rise. This is the same logic as the Baumol effect as applied to capital. We can then calculate a total elasticity of substitution of a component relative to all capital as the logarithmic change in the share divided by the logarithmic change in the relative price.

Table A2 shows the total elasticity for sectors of information capital (as measured by the BEA). The total elasticities are calculated as $\sigma_t = \Delta \ln[s_t/(1-s_t)]/\Delta \ln[p_t/(1-p_t)]$, where s_t is the share of that type of capital in total private fixed assets, and p_t is the price of that asset relative to the price of all private fixed assets. The shaded rows are two principal kinds of assets, information-processing equipment and intellectual property products. It is clear that over the entire 1929–2015 period, the EOSIKs have been greater than unity. This reflects the fact that prices of these types of capital were falling and their shares were rising. The interesting exception is the latest period (2000–2015), in which some forms of information processing equipment showed inelastic substitution (computers and communications equipment).

The summary on the elasticity of substitution for information capital suggests that it has been elastic in production (elasticity of substitution greater than 1). However, the most recent evidence indicates that the elasticity of substitution for the equipment component of information capital is less than unity for the period 2000–2015.

Table A2—Calculated Total Elasticities of Substitution for Different Kinds of Information Capital

Sector	1929–1948	1948–1965	1965–1989	1989–2000	2000–2015	1929–1965	1965–2015	1929–2015
Equipment	1.42	ws	1.49	0.66	0.45	5.20	0.82	1.19
Nonresidential equipment	1.41	ws	1.49	0.67	0.46	5.91	0.83	1.20
Information processing equipment	2.97	4.62	1.70	1.03	0.71	3.68	1.21	1.57
Computers and peripheral equipment	na	na	1.33	1.05	0.60	na	1.14	1.00
Communication equipment	9.72	3.59	2.05	1.08	0.60	4.94	1.11	1.63
Medical equipment and instruments	1.22	ws	6.56	2.26	2.24	2.18	3.58	2.75
Nonmedical instruments	1.76	ws	4.50	2.11	0.84	2.74	2.61	2.68
Photocopy and related equipment	1.32	7.87	1.82	0.04	ws	2.46	0.52	1.24
Office and accounting equipment	0.95	0.87	0.61	ws	ws	0.94	ws	0.04
Intellectual property products	3.39	ws	2.24	6.98	1.28	5.92	2.34	3.46
Nonresidential intellectual property	3.39	ws	2.24	6.98	1.28	5.92	2.34	3.46
Software	na	na	2.65	3.64	1.25	na	2.55	1.00
Research and development	4.90	13.41	2.96	387.32	1.93	6.75	3.17	4.92
Business	5.44	13.12	2.87	ws	1.89	7.23	3.13	5.10
Manufacturing	7.45	13.87	2.06	ws	2.51	9.30	2.63	5.48
Entertainment, etc.	2.69	0.47	ws	46.82	0.57	6.30	0.62	1.97

Note: "na" reflect no data; "ws" or wrong sign reflect negative elasticities.

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